Journal of Engineering Physics and Thermophysics, Vol. 65, No. 2, 1993

A STEFAN-TYPE PROBLEM CONCERNING LIQUID FILTRATION IN A CRACKED-POROUS BED

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Approximate solutions are constructed for a problem on the motion of a crack closure front in an elastically deformed cracked-porous bed.

A strong nonlinear dependence of effective parameters of a medium on its stress state and the liquid pressure is characteristic of filtration processes in cracked-porous media [1]. Therefore, with decreasing bottom pressure near a well, a zone with closed cracks [2] with a moving front can appear.

The problem of the motion of a crack closure front was considered for the first time in [3]; there, the authors used one equation for the Leibenzon function with different bed parameters before and after closure of cracks. A quasistationary analysis of this process within the framework of a modification [4] of the interpenetrating continua model [5] has been carried out in [6]. In the present work we have constructed asymptotic solutions to the problem of the motion of a crack closure front for the case of a deformable medium under the conditions of hydrostatic uniform compression with stress σ. The solutions are also valid with arrangement of all the cracks in one plane (this is typical for sedimentary rocks); here σ is the stress normal to this plane.

1. The filtration process within the zone of open cracks is described in dimensionless form by the equations [4]

$$a\frac{\partial\varphi_1}{\partial t}=g\Delta\varphi_1^n+\varphi_2^+-\varphi_1,\quad \frac{\partial\varphi_2^+}{\partial t}=\varepsilon g\Delta\varphi_2^+-\varphi_2^++\varphi_1,$$
$$\varepsilon\ll 1,\ n=4,\ 0<\varphi_1\leqslant 1,\ \varphi_2\leqslant 1. \tag{1.1}$$

At $\varphi_1 \le 0$ the cracks are closed and liquid filtration occurs only over blocks in accord with the equation

$$\frac{\partial\varphi_2^-}{\partial t}=\varepsilon g\Delta\varphi_2^-. \tag{1.2}$$

At the boundary $x_* = x_*(t)$ between the zones of open and closed cracks the conjugation conditions [7]

$$\varphi_1=0,\ \varphi_2^-=\varphi_2^+,\ \frac{\partial\varphi_2^-}{\partial x}=\frac{\partial\varphi_2^+}{\partial x},\ \frac{\partial\varphi_1^n}{\partial x}=0. \tag{1.3}$$

are satisfied. The last two conditions signify here the equality of flows over the blocks from the left and the right of the front x_* and nonseepage of liquid through the cracks.

We restrict our consideration below to the case of plane-parallel filtration to a gallery.

We pass to a system of coordinates fixed at the moving front of crack closure by means of the substitution

$$\xi=x-ct. \tag{1.4}$$

The order of magnitude of c in (1.4) is important for subsequent considerations.

In the stationary case, the small parameter ε in (1.1) indicates [7] the existence of a boundary layer near the crack closure front. It is natural to expect that the boundary layer exists also during front motion, and on the

Ural State University, Ekaterinburg. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 65, No. 2, pp. 207-213, August, 1993. Original article submitted September 9, 1991.

1062-0125/93/6502-0785$12.50

basis of this we may use asymptotic methods for constructing the solution [8]. In the case of finite values for c, after substitution of (1.4) in (1.1) and (1.2), it follows from the equations for internal expansion of the solutions, obtained as a result of the substitution

$$\xi = \eta\varepsilon^{1/2}, \tag{1.5}$$

that in the first approximation $d\varphi_2^+/d\eta = 0$. Thus, liquid filtration over the blocks at the crack closure front is found to be negligibly small; this finding contradicts the essence of the process under consideration. In addition, a comparison between the results of a depletion process simulation [3], based on introduction of the crack closure front, and known experimental data for a number of deposits has shown that the front moves rather slowly. Relying on this fact, we may assume in (1.4) that

$$c \doteq c_0\varepsilon^{1/2}. \tag{1.6}$$

As a result of the substitution (1.4), under condition (1.6), Eqs. (1.1) and (1.2) take the form

$$-ac_0\varepsilon^{1/2}d\varphi_1/d\xi = gd^2\varphi_1^n/d\xi^2 + \varphi_2^+ - \varphi_1,\ \xi > 0, \tag{1.7}$$

$$-c_0\varepsilon^{1/2}d\varphi_2^+/d\xi = \varepsilon gd^2\varphi_2^+/d\xi^2 - \varphi_2^+ + \varphi_1,\ \xi > 0, \tag{1.8}$$

$$-c_0d\varphi_2^-/d\xi = \varepsilon^{1/2}gd^2\varphi_2^-/d\xi^2,\ \xi_0 < \xi \leqslant 0, \tag{1.9}$$

where ξ_0 = -ct is the location of the gallery with respect to the crack closure front at the moment t. We add immediately that for a finite bed with length l the location of its external boundary relative to the front at the moment t is $\xi_1 = l$ - ct. Conditions (1.3) are fulfilled now at $\xi = 0$.

The system of equations (1.7), (1.8) has the first integral

$$\varepsilon^{1/2}c_0(\varphi_2^+ + a\varphi_1) + g\left(\frac{d\varphi_1^n}{d\xi} + \varepsilon\frac{d\varphi_2^+}{d\xi}\right) = A, \tag{1.10}$$

which is used below instead of (1.7).

2. We consider filtration in the infinite bed. For the initial pressure $\varphi^0 = 1$ we find from (1.10)

$$A = \varepsilon^{1/2}c_0(1 + a).$$

The solution to the system of equations (1.8), (1.10) is constructed by the method of joined asymptotic expansions [8]. The zeroth approximation for the external expansion has the form

$$\varphi_1 = \varphi_2^+ = B, \tag{2.1}$$

where B is a constant.

To construct the internal expansion, we substitute (1.5) into (1.8) and (1.10). Equation (1.8) takes the form

$$g\frac{d^2\varphi_2^+}{d\eta^2} + c_0\frac{d\varphi_2^+}{d\eta} - \varphi_2^+ + \varphi_1 = 0. \tag{2.2}$$

From (1.10), using the last of conditions (1.3), in the first approximation we find $d\varphi_1^n/d\eta = 0$. In the second approximation we obtain the equation

$$gd\varphi_2^+/d\eta + c_0(\varphi_2^+ + a\varphi_1) = c_0(1 + a). \tag{2.3}$$

The solution to the system of equations (2.2), (2.3) has the form

$$\begin{gathered}\varphi_1 = 1 + C_1\alpha_1\exp(\lambda_1\eta) + C_2\alpha_2\exp(\lambda_2\eta),\\ \varphi_2^+ = 1 + C_1\exp(\lambda_1\eta) + C_2\exp(\lambda_2\eta),\\ \alpha_i = -\frac{1 + g\lambda_i/c_0}{a},\ i = 1,\ 2,\end{gathered} \tag{2.4}$$

$$\lambda_{1,2}=-\frac{b_1}{2}\pm\left[\left(\frac{b_1}{2}\right)^2+b_2\right],\quad b_1=\frac{c_0}{g}-\frac{1}{ac_0},\quad b_2=\frac{a+1}{ag},$$

where $\lambda_1>0$, $\lambda_2<0$.

From the conditions of joining (2.1) with (2.4) we find B = 1, C_1 = 0. Using the conjugation conditions (1.3), we obtain

$$C_2=-\alpha_2^{-1},\ D=g\lambda_2/(c_0\alpha_2),\ E=1+a.$$

The solution of Eq. (1.9) has the form

$$\varphi_2^-=E+D\exp[-c_0\xi/(g\varepsilon^{1/2})]. \tag{2.5}$$

Arguments similar to those given above can be made for any initial bed pressure φ^0. In this case the solutions tor the system of equations (1.8), (1.10) and Eq. (1.9) are written in the final form

$$\begin{gathered}\varphi_1(\xi)=[1-\exp(\lambda_2\xi/\varepsilon^{1/2})]\varphi^0,\\ \varphi_2^+(\xi)=[1-\alpha_2^{-1}\exp(\lambda_2\xi/\varepsilon^{1/2})]\varphi^0,\\ \varphi_2^-(\xi)=[1+a+(g\lambda_2/(c_0\alpha_2))\exp(-c_0\xi/(g\varepsilon^{1/2}))]\varphi^0.\end{gathered} \tag{2.6}$$

The velocity of motion of the crack closure front is found from the condition at the gallery

$$\varphi_2^-=\varphi_0,\ \xi=\xi_0 \tag{2.7}$$

or

$$\varepsilon g d\varphi_2^-/d\xi=q,\ \xi=\xi_0. \tag{2.8}$$

In the general case it is impossible to obtain an analytical expression for the velocity. For a << 1 and under the condition (2.7) we have

$$c_0^2=gt^{-1}\ln(1-\varphi_0/\varphi^0). \tag{2.9}$$

With increase in time the motion of a crack closure front in an infinite bed is retarded. The law of front motion for a << 1 in the first approximation has the form

$$x_*^2=g\varepsilon t\ln(1-\varphi_0/\varphi^0). \tag{2.10}$$

Analogously, $x_*^2\sim\varepsilon t$ in the quasistationary solution [6]; however, the dependence of x_* on the bottom pressure, in contrast to (2.10), is a directly proportional one.

We consider the moment of crack closure at the gallery. From (2.6) and (1.3) we find

$$\varphi_2^-(0)=1-\alpha_2^{-1}<0,$$

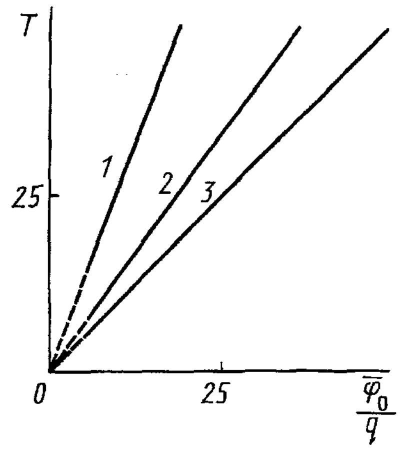
which is easy to verify. At the same time at the gallery we must fulfill the condition $\varphi_2^+(0)=\varphi_1(0)\geq 0$. Physically this means that at the moment of formation of the crack closure front at the gallery the bottom pressure falls abruptly; this fact agrees with the numerical calculation results [9].

3. We consider the depletion process for a unit-dimensional bed when the gallery operates with the constant debit q. The constructed solutions (2.6) satisfy the depletion conditions

$$d\varphi_1/d\xi=d\varphi_2^+/d\xi=0 \tag{3.1}$$

only at $\xi\to\infty$. We try to use solutions of the form (2.6) for the depletion process simulation for the finite bed, satisfying (3.1) approximately and assuming that

$$d\varphi_2^+/d\xi=\gamma\ll 1,\ \xi=\xi_1, \tag{3.2}$$



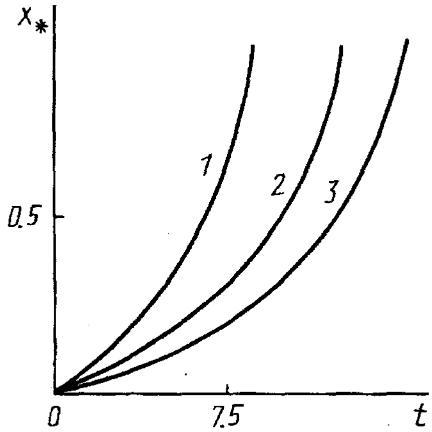


Fig. 1. Time for the front to traverse the deposit; g = 1; ε = 0.01: a = 0.1 (1), 1 (2), 10 (3).

Fig. 2. Motion of the crack closure front under depletion conditions; g = 1; ε = 0.01: a = 0.1 (1), 1 (2), 10 (3).

where ξ_1 is the bed contour location with respect to the crack closure front. Since from (2.6) it follows that $d\varphi_1/d\xi < d\varphi_2^+/d\xi$ for any ξ, then

$$d\varphi_1/d\xi < \gamma.$$

Using conditions (3.2) together with the expression for φ_2^+ from (2.6), we find the expression for the current contour pressure in terms of the initial one

$$\varphi^0 = \delta \exp(-\lambda_2 \xi_1 \varepsilon^{-1/2}),$$
$$\delta = (\overline{\varphi}_0 \overline{C}_2 \overline{\lambda}_2/(C_2\lambda_2)) \exp(\overline{\lambda}_2 \varepsilon^{1/2}), \qquad (3.3)$$

where a bar denotes the initial value of the corresponding parameter. From the boundary condition (2.8), formed for the function φ_2^- from (2.6), we obtain, using (3.3),

$$|\xi_0| = -g(\varepsilon^{1/2}\ln d - \lambda_2)(c_0 + g\lambda_2)^{-1},$$
$$d = \delta \varepsilon^{1/2} g \lambda_2 a [q(g\lambda_2/c_0 + 1)]^{-1}. \qquad (3.4)$$

The front reaches the deposit boundary when $|\xi_0| = 1$. Using (3.4), we find the velocity of its motion at this moment

$$c_0 = -g\overline{\lambda}_2, \qquad (3.5)$$

where λ_2 is uniguely defined by the initial conditions of the process.

Using (3.5), the time t = T for the front to traverse the deposit is determined by the expression

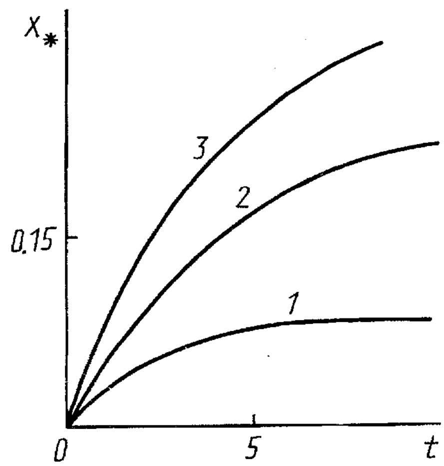
$$T = -(g\varepsilon^{1/2}\overline{\lambda}_2)^{-1}. \qquad (3.6)$$

Omitting the calculations, we give the asymptotics of expression (3.6) for a << 1 in dimensionless time and dimensional time Ω:

$$T \approx \overline{\varphi}^0/q, \quad \Omega \approx L^2\overline{\varphi}^0/(\varkappa_1 q), \qquad (3.7)$$

where L is the dimensional length of the deposit, and κ_1 is the piezoconductivity of the cracks at maximum penetration. Using (3.7), we may estimate the time for the front to traverse the deposit. For example, for L = 500 m, $\kappa_1 = 1$ m^2/sec, and $\varphi^0/q = 10^2$ the time for the front to traverse the deposit is $\Omega \approx 580$ day.

The dependence of the time T for the front to traverse the deposit on the parameter $\overline{\varphi}^0/q$, calculated using formula (3.6), is shown in Fig. 1. With increase in the parameter a, the time T increases. The dashed part of the curves is drawn arbitrarily, because for small values of the parameter $\overline{\varphi}^0/q$, which correspond to low initial contour



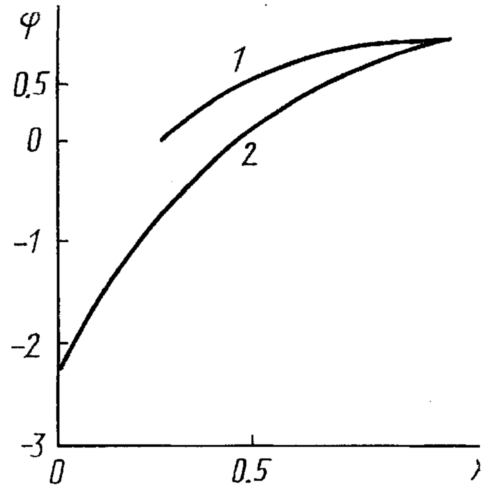


Fig. 3. Motion of the crack closure front; $g = 0.1$; $\varepsilon = 0.01$: 1) head conditions; 2, 3) infinite bed, $a = 10$ (2), 1 (3).

Fig. 4. Pressure distribution over the bed; $g = 1$; $\varepsilon = 0.01$; $c = 0.01$; $\varphi_0 = -2.25$: 1) in cracks; 2) in pores.

pressures or to large bottom debits, the technique presented is not applicable since condition (1.6) ceases to be fulfilled. We note, however, that for a real range of variation in the initial parameters the dimensionless debit values are $q \sim 10^{-2}$-10^{-3}, and the initial contour pressures are $0.1 \le \varphi^0 \le 1$. Thus, $\varphi^0/q \sim 10$-10^2, and there is no meaning considering too small values of the parameter $\overline{\varphi}^0/q$. The numerical calculations given in Fig. 1 were carried out for $\overline{\varphi}^0/q \ge 5$.

Figure 2 shows an example of the motion of the front $x_* = x_*(t)$. As the deposit boundary is approached, the front velocity increases; this is in agreement with the conclusions drawn in [3].

4. Let us consider extraction of liquid from a bed at whose boundary the constant pressure $\varphi^0 = 1$ is maintained. In the integral (1.10) the right-hand side now is

$$A = c_0\varepsilon^{1/2}(1 + a) + gq(\xi_1),$$

where $q(\xi_1)$ is the liquid inflow at the boundary, which has a finite value. Due to this, the zeroth approximation of the external expansion for the solution to the system of equations (1.8), (1.10) has instead of (2.1) the form

$$\varphi_1 = \overset{+}{\varphi}_2 = (q_1\xi + B)^{1/n}, \quad q_1 = q(\xi_1). \tag{4.1}$$

The internal expansion of the solution in the form (2.4) does not join with (4.1), and therefore, the procedure of its construction must be changed. Retaining the correspondence to the physical process and taking into account that near the closure front the pressure in the cracks is small, we shall seek the zeroth approximation of the internal expansion in the form

$$\varphi_1 = \varphi_{10}\varepsilon^{1/n}, \quad \overset{+}{\varphi}_2 = \overset{+}{\varphi}_{20}, \tag{4.2}$$

where φ_{10}, $\overset{+}{\varphi}_{20}$ have finite values.

Omitting the calculations, we present the final form of the solutions to the system of equation (1.8), (1.10):

$$\begin{gathered}
\varphi_1^n = q_1\xi + B_1 - C_2\left(c_0/(g\lambda_2) - 1\right)\exp\left(\lambda_2\xi/\varepsilon^{1/2}\right), \\
\overset{+}{\varphi}_2 = (q_1\xi + B)^{1/n} + C_2\exp\left(\lambda_2\xi/\varepsilon^{1/2}\right), \\
\lambda_2 = -c_0/(2g) - \left[(c_0/(2g))^2 + g^{-1}\right]^{1/2}, \quad 0 < \xi < \xi_1.
\end{gathered} \tag{4.3}$$

The solution to Eq. (1.9) is described as before by expression (2.5). The integration constants are found from boundary conditions $\varphi_1 = \overset{+}{\varphi}_2 = 1$ at $\xi = \xi_1$, conjugation conditions (1.3), and integral (1.10):

$$C_2 = (\beta\xi_1)^{-1},\ \ D = -g\lambda_2 c_0^{-1} C_2,\ \ B = [n(1-\beta^{-1})]^{\frac{1-n}{n}},$$
$$E = g(c_0\varepsilon^{1/2}\xi_1)^{-1},\ \ q = \xi_1^{-1},\ \ B_1 = \varepsilon^{1/2}(\xi_1\lambda_2)^{-1},$$
$$\beta = \varepsilon^{1/2}(c_0 g^{-1} + \lambda_2).$$

From the boundary conditions at the gallery we may obtain equations analogous to (2.9), (2.10) for determining the motion of the crack closure front. Not dwelling on this, we note that at constant bottom and contour pressures the process becomes stationary as $t \to \infty$ and the front has some limiting position $x_* = x^0$. The stationary solution to the system (1.8), (1.10) and Eq. (1.9) is found as the asymptotics of functions (4.3), (2.5) at $c_0 \to 0$, using which we obtain the value of x^0:

$$x^0 = -[(\varepsilon g)^{1/2} + \varepsilon\varphi_0](\varphi_0\varepsilon - 1)^{-1}. \tag{4.4}$$

A value $x^0 > 0$ is reached when $\varphi_0 < (g/\varepsilon)^{1/2}$. The result (4.4) corresponds to the conclusion that in the stationary case the front is removed from the gallery a distance of the order of $\varepsilon^{1/2}$, obtained in [7] in analyzing stationary filtration.

The motion of the front $x_* = x_*(t)$ is shown in Fig. 3. Unlike the depletion conditions, the velocity of the motion decreases with removal of the front from the gallery for an infinite bed and vanishes as $t \to \infty$ under head conditions.

An example of the pressure distribution in the bed under head filtration conditions is presented in Fig. 4.

NOTATION

φ, x, t, dimensionless pressure, coordinate, time; ξ, dimensionless coordinate in the frame of reference associated with the crack closure front; ξ_0, ξ_1, coordinates of the gallery and the bed contour; ε, ratio between the maximum penetrations of cracks and blocks; g, parameter of system (1.1); a, ratio between the elastic capacities of cracks and blocks; c, velocity of the crack closure front; q, dimensionless debit; φ_0, φ^0, bottom and contour pressures; x_*, coordinate of the crack closure front. Subscripts and superscripts: 1, 2) cracks and blocks, respectively, +, -) zones of open and closed cracks.

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